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THE TEMPERATURE FIELD ABOVE A CONCENTRATED HEAT SOURCE ON A VERTICAL ADIABATIC SURFACE

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Abstract—This investigation presents detailed measurements of the laminar temperature field above a highly localized heat source on an instrumented, transparent, vertical adiabatic surface in water. This flow situation is particularly complicated because it is three-dimensional. Small diameter thermocouples were used to measure both the surface temperature variation and the temperature profile in the fluid above the heat source for a range of heat flux. In addition, a Schlieren system was used to visualize the spanwise and normal thermal boundary layers. The downstream growth of the spanwise boundary layer was found to be relatively weak (proportional to $x^{1/5}$), while the boundary-layer thickness normal to the surface above the source was found to increase linearly with downstream distance, x. By comparing these results to those of the well-known point and line source plumes, we conclude that this preferential growth is attributed to the effect of the surface on the entrainment of ambient fluid. Our measurements indicate that the surface temperature above the source decays proportional to $x^{-0.77}$. This is between the x-dependence found for a point source plume (x^{-1}) and a line source of heat on an adiabatic surface ($x^{-3/5}$). Appropriate scaling functions were also determined which collapse measured temperature data for various heat flux and location to a single curve.

NOMENCLATURE

d,	heat source diameter;
g,	acceleration due to gravity;
Gr_d^*,	$\equiv g\beta d^4 q''/k\nu^2$, flux Grashof number;
h,	$\equiv q''/(t_s - t_\infty)$, concentrated source heat-transfer coefficient;
k,	fluid thermal conductivity;
m, M,	constants defined by $\varphi = M\,e^{mx}$;
n, N,	constants defined by $\varphi = Nx^n$;
Nu,	$\equiv hd/k$, Nusselt number based on source diameter;
q'',	heat flux from concentrated heat source;
$t(x, 0, z)$,	spanwise-surface temperature distribution;
$t(x, y, 0)$,	centerplane-normal temperature distribution;
t_0,	$\equiv t(x, 0, 0)$, surface temperature directly above heat source;
t_s,	temperature of heat source;
t_∞,	ambient temperature;
u,	velocity component in x direction;
v,	velocity component in y direction;
w,	velocity component in z direction;
x,	vertical distance above heat source;
y,	distance from heat source normal to surface;
z,	distance from heat source parallel to surface.

Greek symbols

β,	volumetric coefficient of thermal expansion;
$\delta_z(x)$,	spanwise thermal boundary-layer thickness at the surface;
$\delta_y(x)$,	thermal boundary-layer thickness normal to surface in x–y plane;
Δt_0,	$\equiv t_0 - t_\infty$, surface temperature excess directly above source;
Δt_s,	$\equiv t_s - t_\infty$, heat source temperature excess above ambient;
ν,	fluid kinematic viscosity;
φ,	$\equiv \dfrac{t(x,0,0) - t_\infty}{t_s - t_\infty}$, surface temperature excess ratio above heat source;
φ_y,	$\equiv \dfrac{t(x,y,0) - t_\infty}{t_0 - t_\infty}$, centerplane-normal temperature excess ratio;
φ_z,	$\equiv \dfrac{t(x,0,z) - t_\infty}{t_0 - t_\infty}$, spanwise-surface temperature excess ratio.

INTRODUCTION

Many circumstances of transport in the world around us involve highly localized heat sources. Diffusion often arises from a single isolated source. This configuration leads, for example, to a point source (or axisymmetric) plume. Relatively simple laminar flows of this type have been investigated in considerable detail. These, however, are quite simple configurations compared to the complexity of many real situations. For example, when the flow and transport from a point source plume is influenced by even a single bounding surface, it is considerably more complicated, and is not well understood. A principal difficulty is that three dimensional effects enter in. Many real situations are yet more complicated. There may be many bounding surfaces oriented in a complicated way with perhaps the additional influence of multiple and interacting sources. Many examples involving this degree of complexity exist in modern electronic equipment, and in the initiation of some building fires.

Although the problem of proper heat dissipation in electronic equipment has long been considered critical, such transport has received little systematic study in the heat-transfer community. A survey of heat-transfer techniques, as applied to electronic equipment cooling, includes work by Bergles, Chu and Seeley [1]. Other more specific previous investigations have considered overall transport by approximating a collection of heat dissipating devices as either isothermal or uniform flux surfaces. For example, the overall convective cooling of arrays of circuit boards was investigated by Aung, Kessler and Beitin [2]. In their investigation they considered a vertical parallel plate configuration and theoretically and experimentally analyzed the effect of circuit channel height on the maximum circuit board temperature. Their investigation involved the interaction of the convective flows from adjacent circuit boards which were approximated as uniform-heat-flux surfaces.

Additional relevant work on flows between heated vertical plates was done by Aung [3]. He found closed-form solutions for laminar free convection in a vertical, parallel-plate channel with asymmetric heating for both uniform temperature and heat flux surfaces. Aung, Fletcher and Sernas [4] considered nondeveloped flow for the same configuration. All of these previous studies, however, consider overall behavior and do not consider in detail the flow from a single small heat dissipating device which may be approximated as a concentrated heat source. The details of transport from a single source are important, however, in that they determine not only overall operating characteristics but also individual component behavior.

The experiments described here consider a particular aspect of the nature of flows arising from a single concentrated heat source on a vertical adiabatic surface. A schematic showing the coordinate system and flow configuration is shown in Fig. 1. The simpler two-dimensional analogue of the problem considered here is a line source of heat on an adiabatic surface. This has been considered both analytically and experimentally by Zimin and Lyakov [5]. Their experimental results agreed well with the similarity solution predictions when they accounted for the heat loss to the surface. Recent calculations by Jaluria and Gebhart [6] have also considered this problem, for a wider range of Prandtl number. An interesting investigation by Reimann [7] considers another aspect of the two-dimensional analogue of the problem considered here in connection with the Coanda effect and phase interfaces.

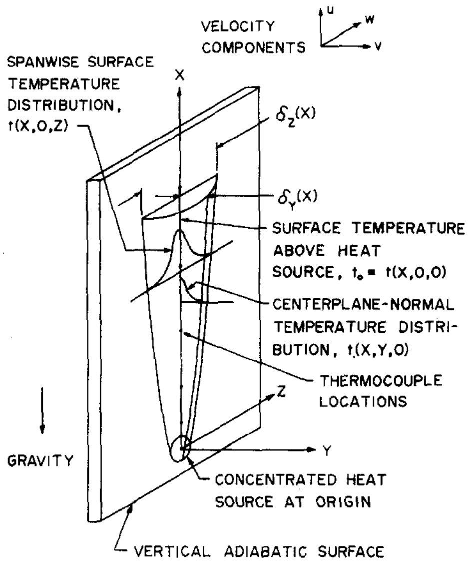

FIG. 1. Coordinate system·and flow configuration.

Motivated by a need to understand the natural convection cooling of electronic devices, Baker [8] considered convection from thin horizontal strips on a vertical surface. He measured higher heat-transfer rates than those calculated using his idealized two dimensional boundary-layer analysis. He suggests that the higher measured heat transfer for thinner strips is due to three-dimensional effects, but he did not examine the details of the flow field. Two-dimensional flow configurations take on relatively little additional physical complexity when a surface is added on a plane of symmetry. This, however, is not the case for an axisymmetric flow configuration.

Even the simplest analytical formulation for constant fluid properties, neglecting viscous dissipation and pressure work and incorporating the two Boussinesq approximations results in a very complicated system of equations. The main complications arise because these circumstances are three dimensional, with the motion-causing buoyancy force bilaterally coupled with viscous, inertia and probably, pressure forces, to the same order. An additional condition for a heat source on an adiabatic surface is that the thermal energy content of the flow remains constant downstream, and is equal to the source input. Configurations of this type have not previously been investigated in detail. The complexity of the governing equations suggested that an experimental study be performed to determine the basic physical characteristics of the resulting downstream mechanisms.

A related and simpler situation is the flow under gravity of fluid originating from a concentrated mass source on an inclined surface. Well-known treatments of these problems are given by Batchelor [9] and Watson [10], and later by Clarke [11], Ackerberg [12] and Smith [13]. More recent work by Merkin [14] considers the separation of such flows on a variable incline. Another significant recent contribution is a similarity solution of the same problem by Smith [15]. Unfortunately, the analytical methods of [15] are not successful when the flow is driven by thermal buoyancy.

In the present study, some of the important characteristics of the temperature field above a concentrated heat source were determined. A Schlieren optical system was used to observe the temperature field above a concentrated heat source adjacent to a surface that very closely approximated adiabatic conditions in water. This study was done in water because it is generally much more difficult to approximate an adiabatic condition in gases than in liquids. Heat fluxes from the concentrated heat sources used in this investigation ranged from 5 to 7 W/cm^2. Transistors may typically dissipate up to about 3 W/cm^2 depending on size. The surface temperature decay above the source was measured using thermocouples embedded slightly beneath the surface. In addition, a thermocouple probe was used to determine temperature profiles in the fluid above the heat source. The spanwise (z direction) and normal (y direction) growth of the thermal boundary layer were then determined from the thermocouple measurements. The present experimental study provides some of the much-needed physical insight necessary for possible analytical simplifications. For example, it was not clear prior to this investigation, how the surface temperature above the source decayed or how the flow spread in the normal (y) and/or spanwise (z) directions. Details of the experimental apparatus and instrumentation are described in the next section.

EXPERIMENTAL APPARATUS

Design considerations led us to preliminary experiments concerning the mechanics and flow region extent. Based upon these initial results we designed and built an adequate size surface instrumented with thermocouples and a built-in reference grid. Transparent material was used in order to permit direct visual observation of the spanwise spread of the flow. The surface, approximately 30 cm wide and 60 cm high, was constructed of thin-wall, low thermal conductivity material. It was essentially a plexiglass rectangular box with a thin front surface. The interior contained air and a plexiglass gridwork which both supported the thin front surface and acted as a reference grid. The interior could be evacuated to further reduce surface heat gain. This provision was provided for possible later studies in air. Our calculations showed, however, that evacuation was not necessary for studies in water. A photograph of the surface is shown in Fig. 2. The test section consisted of an insulated 95 l glass-walled tank.

The heat flux from the fluid to the surface was estimated based on the measured surface temperature distribution. These calculations indicated that this surface design approximated an adiabatic condition over the entire flow length (including the heat source) to within about 1.5% in water, see [16], part II. This is about five times better, for the same conditions, than a solid plexiglass sheet 1.27 cm thick. Twelve small (0.0254 mm wire diameter) thermocouples were embedded into the surface to measure both surface and heat source temperatures. This was accomplished by scoring the surface with a knife. The thermocouple lead wires were laid in the resulting groove and a few drops of plexiglass cement were applied. Surface tension drew the cement into the groove, melted the plexiglass locally and produced a thin plexiglass skin over the wires. A thermofoil heater was cemented directly over the lowest (in x) thermocouple bead so that the temperature of the source could be measured. We used special, custom built high watt density (and high wattage) concentrated heat sources which dissipated up to 7 W/cm^2 in water. Both a small (0.47 cm by 0.47 cm) square heat source and a larger (1.3 cm dia) heat source were used. Surface temperature data was taken to downstream distances of about 32 diameters for the larger source and 7 effective diameters for the small source. Because of the much smaller physical size of the temperature field associated with the smaller heat source, and the correspondingly very small temperature differences, most of our data was taken using the larger source. We have, however, some small source data for surface temperature decay above the source which shows the same trends as for the larger source.

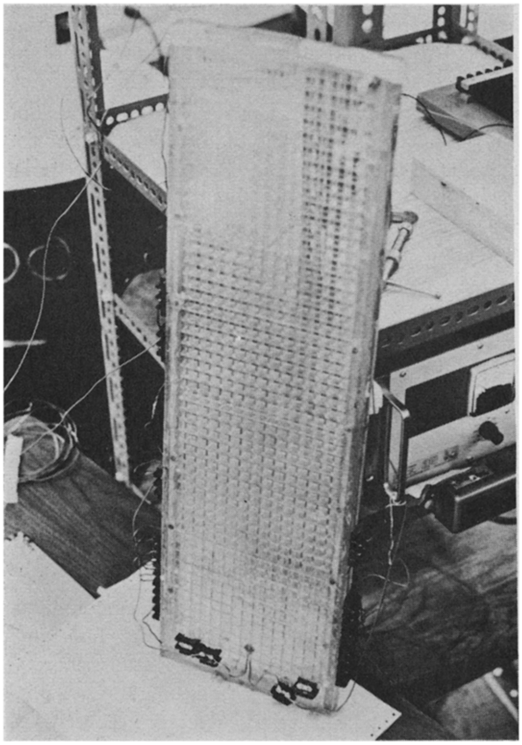

FIG. 2. Photograph of experimental adiabatic surface.

Thermocouple beads were located in the surface along a vertical line above the heat source (the x axis, Fig. 1) and positioned to result in approximately even-spaced temperature data on log–log coordinates. Only the beads were exposed, and the thermocouple lead wires were oriented (slightly subsurface) parallel to the z axis to minimize conduction errors. The thermocouples were referenced to a thermocouple in the ambient fluid and the ambient temperature was measured using a precision thermometer.

The temperature field in the fluid above the source was obtained using a differential thermocouple probe. The thermocouple junction and lead wires were supported horizontally by two small glass tubes held in a plexiglass fixture, which was connected to a micrometer traversing device. The 0.0254 mm diameter thermocouple wire minimized conduction errors as well as disturbance to the flow because of its small size. The error in the thermocouple measurements due to conduction of heat along the lead wires is estimated to be about 0.1% (see [16], part II). Using a micrometer adjustment at the top of the tank, the probe could be positioned with an accuracy of about ±0.0254 mm. The temperature profile along the centerplane of the flow in the y direction, $t(x, y, 0)$, was determined as was the profile at the surface in the z direction, $t(x, 0, z)$. This was done for two heat fluxes and two values of x.

The voltage from the differential thermocouples was measured with a Leeds and Northrup K-4 potentiometer to an accuracy of $\pm 0.5\,\mu$V. The voltage and current to the heat source were measured using a digital voltmeter by measuring the voltage across the foil heater and across a $0.1\,\Omega$ ($\pm 0.00004\,\Omega$) Leeds and Northrup precision resistor in series with the foil. The voltmeter was accurate to ±0.0001 V which provided a measurement of the power dissipation from the heat source accurate to 0.1%.

Because of the low level of density difference in these experiments, we used a newly constructed and very sensitive Schlieren system. The principal optics are 31.75 cm dia first surface parabolic mirrors of 254 cm focal length. The light source diameter is about 2.5 mm. A room 20 m long was used so that very precise collimation and high sensitivity were possible. Interferometric techniques were not used because of the three-dimensional flow field and the associated uncertainties in evaluating interferograms for a density field of initially unknown geometric configuration.

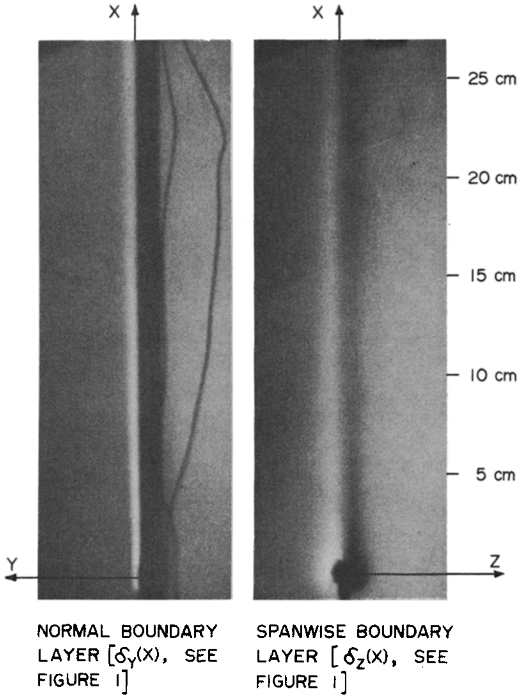

FIG. 3. Schlieren photographs of flow (heat flux: 2.21 W/cm^2).

Because such high Schlieren sensitivity was required to observe the downstream details of these very weak flows, an alternate surface was used for the qualitative flow visualization studies. This was necessary because of optical distortion resulting from very small surface deflection caused by hydrostatic pressure. The surface used for the flow visualization studies consisted of a 1.27 cm thick plexiglass sheet. The heat loss to the solid plexiglass surface was still relatively small (less than 6% of the total over the entire surface; see [16], part II) and the resulting photographs showed the spanwise and normal growth of the boundary layer very well. Schlieren photographs of the flow for the source on this surface were taken at two of the heat fluxes previously investigated with the other surface. All quantitative temperature measurements were taken using the other surface which is adiabatic to within about 1.5%.

NEW MEASUREMENTS AND RESULTS

A schematic of the flow configuration and the corresponding notation is shown in Fig. 1. Using the apparatus and instrumentation described in the previous section, we measured the following physical quantities for a range of heat source strength:

1. Surface temperature decay directly above the source, $t(x, 0, 0) \equiv t_0$.
2. Centerplane-normal temperature distribution, $t(x, y, 0)$, in the fluid.
3. Spanwise-surface temperature distribution, $t(x, 0, z)$.

We also determined the spanwise and normal thermal boundary-layer thicknesses, $\delta_z(x)$ and $\delta_y(x)$, respectively; as well as how the source temperature varied with heat flux.

The main results of these experiments are shown in Figs. 3–8. Figure 3 shows Schlieren photographs of a typical temperature field arising above a concentrated heat source. A vertical knife-edge was used, and the thin black lines on the right side of the left photograph are electrical lead wires. Note also, that in the right photograph, light is transmitted through the transparent adiabatic surface. The first observation in Fig. 3 is that the temperature field normal to the surface, $\delta_y(x)$, is much thinner than the z direction extent, $\delta_z(x)$. Also, the thermal boundary-layer thickness normal to the surface $\delta_y(x)$, is seen to be growing downstream; whereas the spanwise thermal boundary-layer thickness, $\delta_z(x)$, is growing much more slowly. In addition, Fig. 3 shows the progressive decay of the temperature field and suggests the associated weakening of the flow with downstream distance. This characteristic makes fluid temperature measurements increasingly more difficult further downstream. The highly sensitive

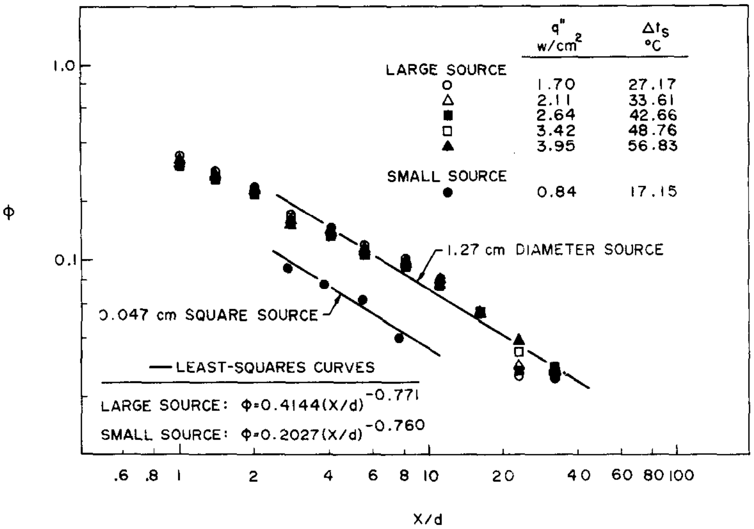

Fig. 4. Variation of dimensionless temperature above the source with non-dimensional downstream distance.

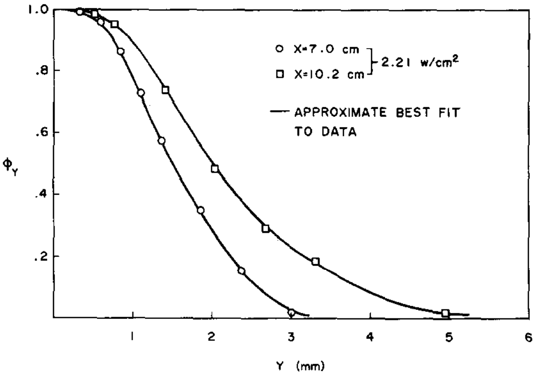

Fig. 5. Typical variation of centerplane temperature excess ratio with surface-normal coordinate (heat flux: 2.21 W/cm^2).

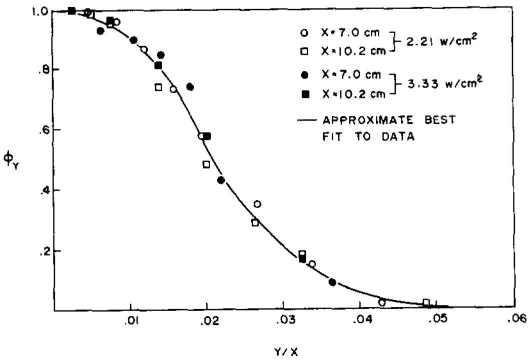

Fig. 7. Variation of centerplane temperature excess ratio with normalized surface-normal coordinate.

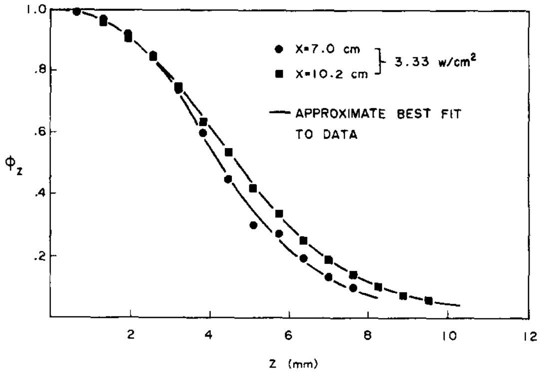

Fig. 6. Typical variation of spanwise-surface temperature excess ratio with spanwise coordinate (heat flux: 3.33 W/cm^2).

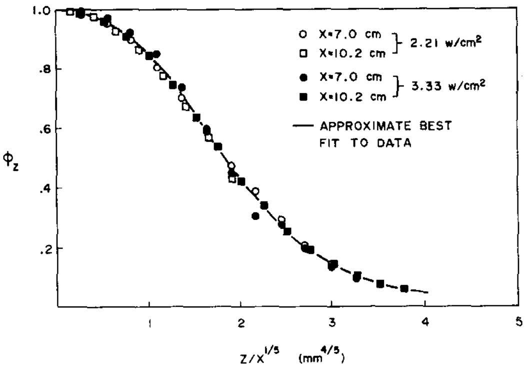

Fig. 8. Variation of spanwise-surface temperature excess ratio with spanwise coordinate.

Schlieren system, however, permitted direct visual observation of the temperature field far downstream, where detailed thermocouple measurements were not possible.

The measured surface temperature directly above the source, $t(x, 0, 0) \equiv t_0$, is shown in Fig. 4 for two source sizes and six heat flux levels. The measured temperature difference between the source and the ambient fluid for each heat flux is also shown. Only one set of data was taken using the small heat source, since the flow was so weak that accurate readings could not be obtained beyond 3.6 cm downstream of the small source ($x/d > 7.57$). The measured values of $\Delta t_0 \equiv t_0 - t_\infty$ beyond $x = 3.6$ cm were quite low ($\Delta t_0 < 0.17°C$) and the repeatability of the data beyond that was not good. For the larger heat source, surface temperature data was taken for five values of heat flux ranging from 1.70 W/cm^2 (for $t_s - t_\infty = 27.2$°C) to 3.95 W/cm^2 (for $t_s - t_\infty = 56.8$°C). This corresponds to a range of total heat dissipation of from 2.15 to 5.00 W. The limit for the maximum heat flux was chosen to prevent boiling on the surface of the thermofoil heater. The minimum heat flux was chosen to ensure temperature differences that were large enough to measure accurately. The resulting data is shown in non-dimensional form in Fig. 4.

Since the functional form of t_0 for this flow configuration was not known, *a priori*, the data was also plotted on a semi-log plot (see [16], part II). In addition, the data for each heat flux was analyzed using a curve-fitting computer program. From the data, the program calculated the least-squares fit of the two functional forms: exponential ($\varphi = M\,e^{mx}$) and power-low ($\varphi = Nx^n$). Data points closer than 2.8 source diameters ($x/d < 2.8$) were not included in this analysis to ensure that the data used to evaluate the centerline decay of the flow included only those points in the fully developed region of the flow. The value of 2.8 was chosen because there is a distinct change in slope of the data on a rectilinear plot at this downstream location. Along the heat source, the thermal boundary condition is one of uniform heat flux, and the temperature field requires some distance to adjust to the adiabatic wall condition imposed downstream of the source.

The results of the curve-fitting analysis for the small source data indicates that the surface temperature above the source could be either power-law or exponential in form. The exponential form fits the data slightly better than the power-law form, but the difference is not great. For the larger heat source, the analysis of the data indicates that the power-law form fits the data slightly better than the exponential form. Again the difference is so slight that it is difficult to explicitly define the form of the centerline temperature decay. We note, however, that for the power-law form the value of n obtained for the large source and the small source are essentially the same ($n = -0.760$ for the small source, $n = -0.771$ for the large source). For the exponential form, the value of m obtained for the small source is quite different from the value obtained for the large source ($m = -0.164$ for the small force, $m = -0.0635$ for the large source). This suggests that the power-law form is preferable, based on the assumption that far enough away from a concentrated heat source, the centerline temperature should decay in a manner that is independent of the source size.

The precedent for this assumption can be found in other natural-convection flows. Far above a heated wire, the natural convection flow closely approximates the theoretical flow from a line source, regardless of the wire size. Likewise, far from the source, the convective flow from a heated sphere closely approximates the theoretical flow from a point source of heat. The different values of m obtained with the different source sizes implies that the centerline decay rate depends on the source size. This is inconsistent with the observed behavior of axisymmetric and plane thermal plumes. For the two source sizes considered, the values of n are almost identical, implying that the form of the surface temperature decay above the source is not dependent on the source size. This is consistent with the trends observed for other convective flows. Therefore, from our analysis of the surface temperature data above the source, we conclude that the temperature decays proportional to $x^{-0.77}$. It is significant that the exponent of the x dependence we found is nearly exactly between that of a point-source thermal plume (proportional to x^{-1}) and that of a line source of heat on a vertical adiabatic surface (proportional to $x^{-3/5}$).

Physically this seems quite reasonable. The temperature decay downstream is a consequence of heat and momentum diffusion. There is an associated growing of the flow region, and an increase in the local mass flow rate as colder fluid is entrained. From the well-known similarity solutions for line and point source plumes (of constant energy content), it can be shown that the temperature above the source is inversely proportional to the local mass flow rate. In addition, the flow region thickness is inversely proportional to the entrainment velocity. The faster cooling of axisymmetric plumes is consistent with the fact that axisymmetric flows permit cooler ambient fluid to be entrained in all radial directions, converging on the plume axis. However, when a vertical adiabatic surface is imposed above a point source plume, one-half of the fluid medium previously available for entrainment is disallowed. Thus it is not surprising that the presence of a surface results in a slower temperature decay. There is, nevertheless, some focusing (toward the x-axis) of entrained ambient fluid for the flows considered here. However, there is no such focusing, in two-dimensional flows, when the direction of the entrainment velocity is normal to the centerplane of the flow. This is consistent with our measurements which indicate a more rapid temperature decay than for similar planar flows above a line source of heat.

Figure 5 shows a typical measured nondimensional centerplane temperature variation, φ_y, across the flow region at two downstream locations ($x = 7.0$ and 10.2 cm) for an energy input rate of 2.21 W/cm^2. The two x locations were chosen far enough downstream to insure measurements in the fully developed region of the flow. On the other hand, the rapidly decaying temperature field (with downstream distance) restricted our measurements to moderate values of x. This data was taken using a 1.3 cm dia heat source, and the temperature profiles were determined using the differential thermocouple probe described in the previous section. A distinct x-dependence is shown. Figure 6 shows a typical spanwise surface temperature distribution, $t(x, 0, z)$, at the same downstream locations and for 3.33 W/cm^2. The spanwise surface temperature variation was inferred from traverses in the z direction (at fixed x) and at $y \approx 0.254$ mm. These measurements were compared with the temperatures measured by the thermocouples in the surface (at the same x and at $y = z = 0$). The temperatures agreed to within a maximum deviation of 0.28°C at temperature difference levels of about 17°C. This, and the measured small fluid temperature gradients near the surface permits inference of the spanwise surface temperature variation.

The x-dependence shown in Fig. 6 is seen to be much weaker than that shown in Fig. 5 indicating a preferential growth of the temperature field in the direction normal to the surface. This is consistent with the Schlieren photographs shown in Fig. 3. In fact, a closer examination of Fig. 3 (using a straight-edge) suggests that the normal thermal boundary-layer grows about linearly with x. This indicates the possibility of normalizing the data with a characteristic length proportional to x. To further explore this, the (x, y) coordinates of several pairs of data points corresponding to constant values of φ_y were analyzed. For each pair of points, the average value of the exponent of the x variation was found to be 1.01. Therefore, the data of Fig. 5 as well as the corresponding data for a heat flux of $3.33\,\mathrm{W/cm^2}$ are shown plotted against y/x in Fig. 7. It can be seen that this normalization collapses the data to a single curve quite well.

Similarly, it can be seen, both from Fig. 6 and the Schlieren photographs, that the spanwise growth of the boundary layer is a very weak function of x. This suggests a normalization with a characteristic length that is an appropriately weak function of x. To determine what power of x would achieve this, pairs of points at constant φ_z were analyzed as previously mentioned. The average exponent of the x-dependence was found to be 0.21. Therefore, the data of Fig. 6 as well as the corresponding data for a heat flux of $2.21\,\mathrm{W/cm^2}$ is shown plotted against $z/x^{1/5}$ in Fig. 8. It is clear that this collapses the data onto essentially a single curve. It further implies that the spanwise boundary layer grows about proportional to $x^{1/5}$.

CONCLUSIONS

The results of the thermocouple measurements in this study indicate that the surface temperature, t_0, directly above a concentrated heat source follows a power-law variation with downstream distance. The exponent of the x-dependence was found to be -0.77. It is interesting to note that this variation of t_0 with x is between that downstream of a horizontal line-source of heat on an adiabatic surface ($t_0 - t_\infty \propto x^{-3/5}$), and in a freely rising axisymmetric plume ($t_0 - t_\infty \propto x^{-1}$) from a point source. The flow studied here cools faster than a plane flow, due to enhanced entrainment, but not as rapidly as a plume rising from a point source, in the absence of a bounding surface. That is, the surface (in our study) limits the entrainment of cooler fluid from the surroundings, resulting in a weaker centerline temperature decay than for the freely-rising axisymmetric plume.

It is interesting to compare our measurements with the known effect of imposing a vertical, adiabatic surface adjacent to a line source of heat. For these planar plumes, the x-dependence of the temperature decay downstream is unaffected by the presence of a surface. The presence of a surface does, however, change the proportionality constant and causes the flow to cool less rapidly. This is shown quantitatively by Jaluria and Gebhart [6]. For moderate Prandtl number (gases and water), the local temperature difference above the source, $t_0 - t_\infty$, is about a factor of two greater when the same line source of heat is adjacent to a vertical, adiabatic surface. However, for the non-planar flows we considered, our measurements show that the effect of a surface retards the temperature decay in a much more complicated and fundamental way. The x-dependence changes. The measured modification of the x-dependence of t_0 is presumably a consequence of three dimensional effects on heat and momentum diffusion resulting from the imposition of a surface in a previously axisymmetric flow.

Furthermore, it was found that when $t_0 - t_\infty$ was non-dimensionalized with the source temperature difference, $t_s - t_\infty$, the resulting variation was essentially independent of the source heat flux. A source diameter effect, however, was observed. In both cases the temperature decayed about proportional to $x^{-0.77}$, but the proportionality constant was different for each source size considered.

The Schlieren photographs indicate that the thermal boundary layer grows about linearly with x in the direction normal to the surface and very slowly in the spanwise direction. This is consistent with the detailed thermocouple measurements. The data shows a distinct x dependence. Although the fluid temperature measurements were restricted to moderate values of downstream distance, as discussed in the previous section, the Schlieren photographs show the thermal field much further downstream. There is, however, very close agreement between our thermocouple measurements, taken at moderate values of x, and the trends shown in the Schlieren photographs. This permits the inference that the measured temperature profiles are representative, and that their x dependence is the same over the entire range of downstream distances considered. Analysis of this temperature data indicates that the normal boundary-layer thickness, $\delta_y(x)$, grows about linearly with x; whereas the spanwise thickness, $\delta_z(x)$, grows about proportional to $x^{1/5}$. This is consistent with the Schlieren photographs and indicates that the preferential growth direction of the flow is normal to the surface.

These trends were found for all heat flux levels used. We tentatively conclude that the x dependence of characteristic, or scaling lengths, in the y and z directions to be x and $x^{1/5}$, respectively. Data normalized with these characteristic lengths is seen to collapse into a narrow band. Our whole body of experimental results indicated that such normalized temperature profiles were essentially independent of heat flux over the range 3–$7\,\mathrm{W/cm^2}$. In comparison, the point-source plume thermal boundary layer grows proportional to $x^{1/2}$ and that of a line-source plume on an adiabatic surface grows proportional to $x^{2/5}$.

The preferential growth of the thermal boundary layer in the surface-normal direction is consistent with the expected entrainment for such a flow. The entrainment of cooler ambient fluid near the surface is inhibited by the viscous shear force exerted on the incoming fluid by the surface. However, fluid entrained further from the surface is not affected so directly. Consequently at fixed downstream location, the local entrainment velocity at the edge (circumferentially) of the flow region increases with increasing distance from the surface. Therefore, the spanwise boundary layer grows more slowly because of the relatively lower local entrainment velocity there. On the other hand, the relatively higher entrainment velocities at the edge of the flow region, further away from the surface, results in more rapid growth of the boundary layer in the surface-normal direction. Although the thermal boundary-layer temperature profiles were determined for two widely different heat fluxes (2.21 and 3.33 W/cm^2), the effect of heat flux on the profiles appeared to be negligible.

The main objective in the present work has been to determine fundamental characteristics of the temperature field above a concentrated heat source on an adiabatic surface. The results, however, can also be used to determine the heat transfer variation with source temperature. In terms of the Nusselt number, based on source diameter, $Nu \equiv hd/k = q''d/k(t_s - t_\infty)$, and flux Grashof number, $Gr_d^* \equiv g\beta d^4 q''/k\nu^2$, we tentatively conclude that $Nu \propto Gr_d^{*1/4}$, over the range $7.9 \times 10^4 \leqq Gr_d^* \leqq 1.2 \times 10^7$. We have, however, very limited data at lower Gr_d^*, and extensive additional measurements are soon to be reported [17].

The measured fluid temperature profile normal to the surface has a very small slope near the surface, which implies that the heat transfer to the surface is very small. This indicates that the surface used in this study is an excellent approximation to an adiabatic surface. From the measured surface temperature above the source, the heat loss to the surface over the entire length of the surface used for temperature measurements was estimated to be less than 1.5% of the heat flux at the source (see [16], part II).

The results of this study serve to further the understanding of the nature of the thermal boundary layer associated with the buoyancy-induced convection above a concentrated heat source on a vertical adiabatic surface. The thermocouple measurements are consistent both with the observations from the Schlieren photographs and with the conclusions drawn from physical arguments about the flow. It is hoped that the new insight gained will form a basis for further experimental and analytical studies of these types of convective flows.

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CHAMP DE TEMPERATURE AU DESSUS D'UNE SOURCE DE CHALEUR CONCENTREE SUR UNE SURFACE VERTICALE ET ADIABATIQUE

Résumé—Cette étude présente en détail des mesures du champ de température au dessus d'une source de chaleur très localisée sur une surface transparente, verticale et adiabatique dans l'eau. Le type d'écoulement est particulièrement compliqué par sa tridimensionnalité. On utilise des thermocouples de petit diamètre pour mesurer à la fois la variation de la température de surface et le profil de température dans le fluide au dessus de la source de chaleur pour une gamme de flux thermique. De plus un système Schlieren est utilisé pour visualiser les couches limites dans leur étendue et leur épaisseur. La croissance en largeur de la couche limite est relativement faible (proportionnelle à $x^{1/5}$) tandis que l'épaisseur normalement à la surface, au dessus de la source, croît linéairement en fonction de x. En comparant ces résultats à ceux des panaches de sources ponctuelles et linéaires, on conclut que cette croissance préférentielle est attribuable à l'effet de la surface sur l'entrainement du fluide ambiant. Les mesures montrent que la température de surface au dessus de la source décroît proportionnellement à $x^{-0,77}$. Ce cas se situe entre la dépendance en x du panache de source ponctuelle (x^{-1}) et celle d'une source linéaire sur une surface adiabatique ($x^{-3/5}$). Des fonctions sont déterminées qui rassemblent en une courbe unique les mesures de température pour de nombreux flux thermiques et plusieurs positions.

DAS TEMPERATURFELD OBERHALB EINER KONZENTRIERTEN WÄRMEQUELLE AN EINER VERTIKALEN, ADIABATEN OBERFLÄCHE

Zusammenfassung—Es wird über detaillierte Messungen des laminaren Temperaturfeldes oberhalb einer stark konzentrierten Wärmequelle auf einer mit Meßstellen versehenen transparenten, vertikalen, adiabaten Platte in Wasser berichtet. Die Strömung ist dreidimensional und deshalb besonders kompliziert. Über Thermoelemente mit geringem Durchmesser werden sowohl die Temperaturverteilung in der Oberfläche wie im Fluid oberhalb der Wärmequelle für verschiedene Wärmeströme gemessen. Mit Hilfe des Schlierenverfahrens wurden die thermischen Grenzschichten in Quer- und Längsrichtung sichtbar gemacht. Für die Grenzschicht in Querrichtung ergab sich nur ein relativ schwaches Anwachsen in Strömungsrichtung (proportional $x^{1/5}$); die Grenzschichtdicke oberhalb der Wärmequelle wächst dagegen linear mit dem Abstand x von der Wärmequelle in Strömungsrichtung an. Aus einem Vergleich dieser Ergebnisse mit den bekannten Resultaten für punkt- und linienförmige Wärmequellen wird geschlossen, daß das bevorzugte Wachstum durch den Einfluß der Oberfläche auf das Mitreißen von Umgebungsflüssigkeit bedingt ist. Die eigenen Messungen zeigen einen zu $x^{-0,77}$ proportionalen Abfall der Oberflächentemperatur in Strömungsrichtung oberhalb der Wärmequelle. Dieser Wert liegt zwischen demjenigen für eine punktförmige Wärmequelle (x^{-1}) und dem für eine linienförmige Wärmequelle ($x^{-3/5}$) auf einer adiabaten Oberfläche. Durch geeignete Maßstabsfunktionen können die gemessenen Temperaturen für verschiedene Wärmeströme mit Hilfe einer einzigen Kurve dargestellt werden.

ТЕМПЕРАТУРНОЕ ПОЛЕ НАД СИЛЬНО ЛОКАЛИЗОВАННЫМ ИСТОЧНИКОМ ТЕПЛА НА ВЕРТИКАЛЬНОЙ АДИАБАТИЧЕСКОЙ ПОВЕРХНОСТИ

Аннотация — Представлены подробные результаты измерений ламинарного температурного поля над сильно локализованным источником тепла, расположенным на снабженной датчиками прозрачной вертикальной адиабатической поверхности, погруженной в воду. Картина течения является весьма сложной в силу трехмерности задачи. Термопарами небольшого диаметра замерялось как изменение температуры самой поверхности, так и температуры жидкости над источниками тепла в исследуемом диапазоне значений плотности теплового потока. Визуализация картины распространения тепловых пограничных слоев вниз по пластине и по нормали к ней осуществлялась методом теневого фотографирования. Найдено, что вниз по пластине пограничный слой распространяется довольно медленно, (пропорционально $x^{1/5}$), в то время как толщина пограничного слоя нормально к поверхности над источником тепла увеличивается линейно с расстоянием x. Сравнение полученных результатов с данными по конвективным струйкам над точечными и линейными источниками позволило заключить, что этот избирательный рост толщины слоя объясняется увлечением поверхностью окружающей жидкости. Проведенные измерения показали, что температура поверхности над источником уменьшается пропорционально $x^{-0,77}$. Эта зависимость лежит между зависимостями для конвективных струек над точечным источником (x^{-1}) и над линейным источником тепла ($x^{-3/5}$) на адиабатической поверхности. Предложены соответствующие пересчетные коэффициенты, позволяющие описать измеренные значения температуры для различных плотностей теплового потока и местоположения источника с помощью одной кривой.